Inviscid Supersonic Far Wake Flow past Pointed Bodies

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Theme

THE flowfield analysis of supersonic decelerators towed in the wake of fast-moving re-entry vehicles is one of significant import to predict their performance characteristics. Since many supersonic decelerators, such as inflatable ballutes and guide surface parachutes, have conical front ends to guide the flow and stabilize the shock ahead of them, simple geometrical configurations such as cones and wedges are considered in the present analysis.

The method of integral relations hitherto applied to blunt bodies in uniform flows is extended to nonuniform far wake flow past pointed bodies. The analysis encompasses both attached and detached shock situations at the apex and includes the transverse variations of wake flow properties and their derivatives ahead of the shock.

Contents

A. Inviscid Flow Model

Figure 1 depicts the flowfield surrounding a primary and secondary body combination. In the near wake, coalescing of free shear layers that enclose an inner recirculating flow and turning of the flow in the vicinity of the rear stagnation point form a wake neck. In the early part of the far wake region, large longitudinal flow variable gradients produced by the shock at the primary body nose and the recompression shock at its base create an outer, wider inviscid entropy wake enclosing an inner, narrower viscous wake.

Eventually, far downstream the longitudinal flow variable gradients decay with flow profiles approaching asymptotic shapes. In addition, the viscous dissipation becomes negligible, suggesting that the far wake flow may be treated as an inviscid flow. It is characterized by a nonuniform variation of Mach number, total pressure, entropy, and flow direction. The far downstream locations of the secondary body are favorable for deceleration purposes since it becomes aerodynamically stable and produces more drag.

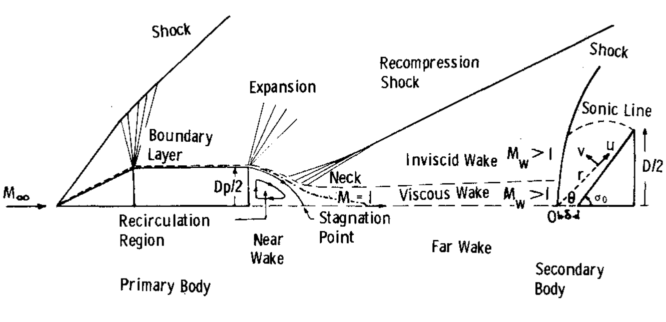

Fig. 1 Flowfield surrounding a two-body system.

B. One-Strip Approximation

In applying the method of integral relations, the flow equations, written in a polar coordinate system with its origin at shock-symmetry point, must be cast in the divergence form

$$(\partial/\partial r)F_i(r,\theta,u,\,..) + (\partial/\partial\theta)G_i(r,\theta,u,\,..) + H_i(r,\theta,u,\,..) = 0 \quad i = 1,2 \tag{1}$$

where

$$\begin{aligned} F_1 &= \rho u \exp(S/R) r^{\nu+1}\sin^\nu\theta & F_2 &= \rho u v r^{\nu+1}\sin^\nu\theta \\ G_1 &= \rho v \exp(S/R) r^\nu \sin^\nu\theta & G_2 &= (p+\rho v^2) r^\nu \sin^\nu\theta \\ H_1 &= 0 & H_2 &= \rho u v r^\nu \sin^\nu\theta - \nu p r^\nu \cos^\nu\theta \end{aligned} \tag{2}$$

and $i = 1,2$ correspond to entropy-continuity and θ-momentum equations, respectively. The independent variable θ will be normalized using the relation

$$\eta = (\theta-\theta_b)/(\theta_s-\theta_b) \qquad \nu = 0, \text{two-dimensional flow}$$
$$\eta = (\cos\theta - \cos\theta_b)/(\cos\theta_s - \cos\theta_b) \qquad \nu = 1, \text{axisymmetric flow}$$

so that $\eta = 0$ and 1 correspond to body and shock surfaces, respectively.

In one-strip approximation, the Eq. (1) is integrated with respect to η from the body surface to shock wave. Application of the Leibnitz rule and a linear representation of the unknown integrand functions $f[=\rho u v \exp(S/R)]$ and $g(=\rho u v)$ reduces these equations to a first-order system of ordinary differential equations. For a nonuniform far wake flow ahead of the shock, the functions f_s and g_s can be expressed in terms of the flow variables in the wake. The functional form of f_s, for example, will be

$$f_s = f_s[\theta_s, \beta, P_{T1}, M_1, (S/R)_1, \varepsilon]$$

Considering the mass flux $Q(=\rho_b V_b)$ and the shock angle β as dependent variables, the integrated equations can be reduced to the form

$$\exp(S/R)_b \cos(\theta_b - \sigma) r(dQ/dr) + B_1 r(d\beta/dr) + C_1 = 0 \tag{3}$$

$$-V_b \sin[2(\theta_b - \sigma)] r(dQ/dr) + B_2 r(d\beta/dr) + C_2 = 0 \tag{4}$$

The surface velocity V_b is determined from an algebraic relation

$$V_b[(\gamma-1)/\gamma\ (\phi_{1b} - V_b{}^2/2)/\phi_{2b}]^{1/(\gamma-1)} = Q \tag{5}$$

where ϕ_{1b} and ϕ_{2b} are constants, which are determined from the flow conditions immediately behind the shock at the centerline. The coefficients B_i, C_i $(i = 1,2)$ take into account the variation of the wake flow properties and their derivatives ahead of the shock.[1]

A saddle point type of singularity appears at the sonic point. In the case of a sharp-cornered body, Q must reach its sonic value Q^* at the body corner. If the chosen standoff distance δ is greater than its true value, the computed Q at the body corner is less than Q^*; if smaller, Q reaches Q^* ahead of the corner. Such behavior is exactly the characteristics of a saddle point singularity.

In addition, a differential equation for the shock position angle θ_s can be obtained from the geometry of the shock as

Presented as Paper 73-671 at the AIAA 6th Fluid and Plasma Dynamics Conference, Palm Springs, Calif., July 16–18, 1973; submitted July 9, 1973; synoptic received October 12, 1973; revision received November 19, 1973. Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y., 10017. Price: Microfiche, $1.00; hard copy, $5.00. **Order must be accompanied by remittance.** The author expresses his gratitude to H. G. Heinrich of the University of Minnesota for his interest in this study.

Index categories: Supersonic and Hypersonic Flow; Shock Waves and Detonations; Jets, Wakes, and Viscid-Inviscid Interactions.

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$$r(d\theta_s/dr) = -\tan(\theta_s - \beta) \quad (6)$$

For bodies having a straight generatrix, such as cones and wedges, an algebraic relation $\theta_b = \sigma - \sin^{-1}(\delta/r \sin\sigma)$ determines the body position angle θ_b.

C. Boundary and Initial Conditions

The boundary conditions specified at the ends of a circular arc of radius δ and at the body corner are

$$\text{at} \quad r = \delta, \quad \theta_s = \theta_{s\delta}, \quad \beta = \beta_\delta, \quad Q = 0$$
$$\text{at} \quad r = r^*, \quad Q = Q^* \quad (7)$$

The shock position angle $\theta_{s\delta}$ and shock angle β_δ at $r = \delta$, which are unknown a priori, are determined by a judicious choice of the shock shape in the vicinity of the origin. In the present analysis, these values are obtained by considering a parabolic shock near the origin from $r = 0$ to $r = \delta$.

In the case of an attached shock at the apex of the secondary body, the conditions for the existence of a regular solution at $r = 0$ are $\theta_s = \beta$, $\theta_b = \sigma$, and $C_i = 0$ $(i = 1, 2)$. The nonlinear algebraic equations $C_i = 0$ $(i = 1, 2)$ determine the shock angle β and surface velocity V_b at $r = 0$. Since the initial values are completely specified, the problem appropriately reduces to an initial value type.

D. Numerical Results

Numerical integration is carried out on a CDC 6600 computer using the Runge-Kutta integration method with a variable step size, which is automatically reduced to satisfy a prescribed accuracy. Figure 2 illustrates the results for a 60° half-angle cone in a uniform flow and also in a nonuniform flow, which represents a far wake behind a 26° cone-cylinder at a distance $L/D_p = 11$ from its base. In the uniform flow, the shock shape and pressure distribution compare well with the experimental results of Ref. 2. The effect of nonuniform flow on the shock is to increase its shock layer thickness and to decrease the sonic point distance from the axis. The local pressure coefficient and its gradient are lower for cones in the far wake flow due to the reduced dynamic pressure in the wake.

Application of the preceding analysis for a two-dimensional flow will be found in Ref. 1, where a 35° half-angle wedge (secondary body) is considered in the far wake of a 20° wedge (primary body) with two primary body wall conditions—an adiabatic wall ($T_{\text{wall}}/T_{0\infty} = 0.87$) and a cold wall ($T_{\text{wall}}/T_{0\infty} =$ 0.19). The effect of cooling the primary body is found to increase the subsonic domain of the shock layer. The variation of surface velocity showed the presence of two additional stagnation points and a region of reverse flow. An increase of wedge angle or the flow variable gradients in the wake increases the size of the reverse flow bubble which may eventually enclose the nose region. Such a bubble at the nose of an inflatable decelerator may seriously impair its performance characteristics.

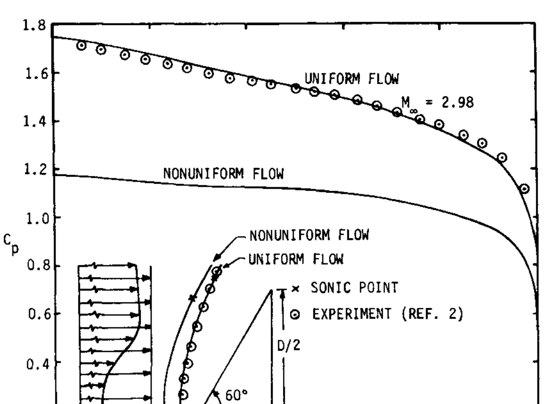

Fig. 2 Comparison of pressure on a 60° half-angle cone in uniform flow at $M_\infty = 2.98$ and in the wake of a 26° cone-cylinder at $M_\infty = 2.98$.

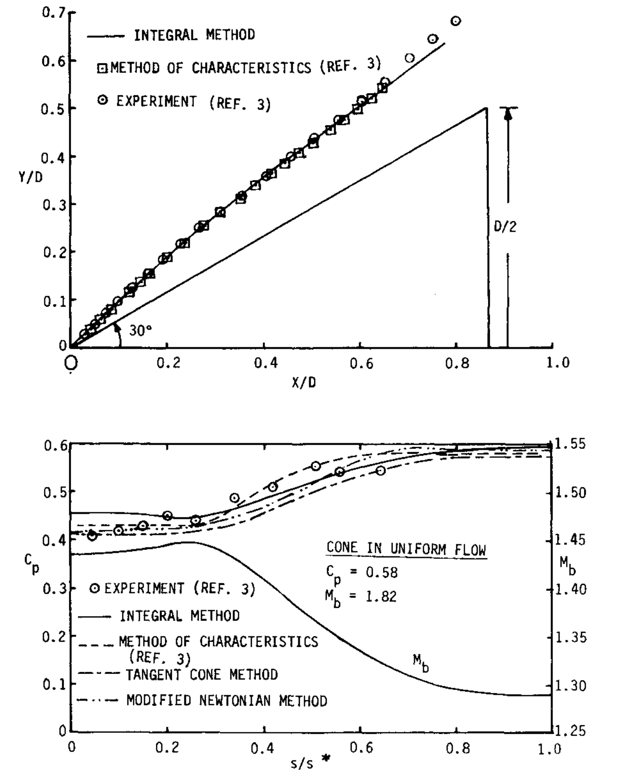

Fig. 3 Shock shape, pressure, and Mach number distributions on a 30° half-angle cone in the far wake ($L/D_p = 9.0$) of the 26° cone-cylinder at $M_\infty = 2.98$.

For an attached shock case, a 30° half-angle cone is considered in the far wake of the 26° cone-cylinder. In Fig. 3, the shock shape compares well with those of experiments and the characteristics method.[3] The computed values of C_p are compared with those of the experiments as well as other analytical methods such as the characteristics, tangent cone, and modified Newtonian methods. Although the surface pressure coefficient approaches its value in the uniform flow, the Mach number attains a value considerably lower than its uniform flow value. The present one-strip integral method adequately predicts the shock shape and pressure distribution for pointed bodies immersed in the supersonic far wake flow.

References

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